Enoncé

Donner la forme algébrique,
pour chacun des nombres complexes suivants.

$$z_1 = (1+i)(1-i)$$

$$z_2 = \frac{2-i}{2+2i}$$

$$z_3 = \left(1+i\sqrt{3}\right)^2$$

$$z_4 = \frac{-1}{i}$$

$$z_5 = \frac{-2+2i}{1-i}$$

Rappel

$$\text{Forme algébrique} : \boldsymbol{z = a + ib} \quad \text{avec} \quad \begin{cases} a = Re(z) \\ b = Im(z) \end{cases}$$

$$\text{Conjugué de } z : \ \boldsymbol{\bar{z} = a - ib}$$

$$\text{Propriété} : \boldsymbol{i^2 = -1}$$

Correction

$$\boldsymbol{z_1 = (1 + i)(1 - i)}$$

$$Identité\ remarquable\ (a + b)(a - b) = a^2 - b^2$$

$$\Longrightarrow\ z_1 = 1 - i^2$$

$$On\ rappelle\ que\ i^2 = -1$$

$$\Longrightarrow\ z_1 = 1 - (-1)$$

$$\Longrightarrow\ z_1 = 2$$

$$avec \quad \begin{cases} Re(z_1) = 2 \\ Im(z_1) = 0 \end{cases}$$

Correction

$$\boldsymbol{z_2 = \frac{2-i}{2+2i}}$$

On applique la ***méthode du conjugué*** :

$$z_2 \text{ est sous la forme } \frac{z_A}{z_B} \implies z_2 = \frac{z_A \cdot \overline{z_B}}{z_B \cdot \overline{z_B}}$$

$$\implies z_2 = \frac{(2-i)(2-2i)}{(2+2i)(2-2i)}$$

$$\implies z_2 = \frac{4-4i-2i+2i^2}{2^2-(2i)^2}$$

$$\implies z_2 = \frac{4-6i-2}{2^2-(2i)^2}$$

$$\implies z_2 = \frac{2-6i}{4-4i^2}$$

$$\implies z_2 = \frac{2-6i}{4+4}$$

$$\implies z_2 = \frac{2-6i}{8}$$

$$\implies z_2 = \frac{2}{8} - \frac{6i}{8}$$

$$\implies z_2 = \frac{1}{4} - \frac{3}{4}i$$

$$avec \quad \begin{cases} Re(z_2) = \frac{1}{4} \\ Im(z_2) = -\frac{3}{4} \end{cases}$$

Correction

$$\boldsymbol{z_3 = \left(1 + i\sqrt{3}\right)^2}$$

$$Identité\ remarquable\ (a+b)^2 = a^2 + 2ab + b^2$$

$$\Longrightarrow\ z_3 = 1^2 + 2i\sqrt{3} + \left(i\sqrt{3}\right)^2$$

$$\Longrightarrow\ z_3 = 1 + 2i\sqrt{3} + 3i^2$$

$$\Longrightarrow\ z_3 = 1 + 2i\sqrt{3} - 3$$

$$\Longrightarrow\ z_3 = -2 + 2i\sqrt{3}$$

$$avec\quad \begin{cases} Re(z_3) = -2 \\ Im(z_3) = 2\sqrt{3} \end{cases}$$

Correction

$$\boldsymbol{z_4 = \frac{-1}{i}}$$

On applique la ***méthode du conjugué*** :

$$z_4 \text{ est sous la forme } \frac{z_A}{z_B} \implies z_4 = \frac{z_A \cdot \overline{z_B}}{z_B \cdot \overline{z_B}}$$

$$\implies z_4 = \frac{(-1)(-i)}{i(-i)}$$

$$\implies z_4 = \frac{i}{-i^2}$$

$$\implies z_4 = \frac{i}{1}$$

$$\implies z_4 = i$$

$$avec \quad \begin{cases} Re(z_4) = 0 \\ Im(z_4) = 1 \end{cases}$$

Correction

$$\boldsymbol{z_5 = \frac{-2+2i}{1-i}}$$

On applique la ***méthode du conjugué*** :

$$z_5 \ est \ sous \ la \ forme \ \frac{z_A}{z_B} \implies z_2 = \frac{z_A \cdot \overline{z_B}}{z_B \cdot \overline{z_B}}$$

$$\implies z_5 = \frac{(-2+2i)(1+i)}{(1-i)(1+i)}$$

$$\implies z_5 = \frac{-2-2i+2i+2i^2}{1^2-i^2}$$

$$\implies z_5 = \frac{-2+2i^2}{1-(-1)}$$

$$\implies z_5 = \frac{-2-2}{2}$$

$$\implies z_5 = \frac{-4}{2}$$

$$\implies z_5 = -2$$

$$avec \quad \begin{cases} Re(z_5) = -2 \\ Im(z_5) = 0 \end{cases}$$